

# Exploring the Effects of Removing Process-Intrinsic Constraints on Gas Turbine Design

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Design by decomposition makes complex design problems tractable. However, decomposition also requires fixing key interdisciplinary design variables, decisions that often lock in much of the final product performance, at the very beginning of the design process. We suggest that a design process based on decomposition thus introduces “process-intrinsic” constraints, in addition to the “hard” constraints driven by physical or customer requirements, which can have important and unexpected implications on the achievable performance in later design stages. Using an integrated design methodology which preserves the inherent multidisciplinary coupling between components and disciplines and does not require assumption of the relevant interactions, the advantages of removing, or “softening,” process-intrinsic constraints can be explored. In this paper we investigate how removing these constraints may improve the adiabatic efficiencies of a high pressure compressor and turbine spool and thus reduce specific fuel consumption. The results indicate that integrating the aerothermal and mechanical analyses can reduce specific fuel consumption by 0.20% but if the process-intrinsic constraints on shaft work, component weight, shaft speed, and bleed air cooling flows are removed a reduction of 0.42% is possible. The cost performance benefit of removing constraints exhibited bimodal behavior. Conservative performance improvements were achieved with and without process-intrinsic constraints but design time decreased as more constraints were removed. More aggressive improvements were achievable only by removing constraints but only with significant computational expense.

## Nomenclature

<i>bezcas1</i>	= compressor casing Bezier control point 1
<i>bezcas2</i>	= compressor casing Bezier control point 2
<i>bezhub1</i>	= compressor hub Bezier control point 1
<i>bezhub2</i>	= compressor hub Bezier control point 2
<i>delr/r</i>	= compressor surge margin
<i>dvexrad</i>	= compressor exit Bezier control point to change exit area
<i>dvdv</i>	= compressor stagewise pressure distribution parameter
<i>HPCMnPk</i>	= compressor peak Mach number
<i>HPCPower</i>	= compressor (spool) power
<i>HPCwt</i>	= compressor weight
<i>HPC<math>\eta</math></i>	= compressor adiabatic efficiency
<i>HPTANsq</i>	= turbine midheight rotor blade area $\times$ blade rotational speed squared
<i>HPTARR</i>	= turbine rotor blade aspect ratio
<i>HPTARV</i>	= turbine nozzle blade aspect ratio
<i>HPTalph2</i>	= turbine nozzle exit flow angle
<i>HPTBrel2</i>	= turbine nozzle exit flow angle relative to rotor
<i>HPTCLfrR</i>	= turbine rotor optimum lift coefficient fraction
<i>HPTCLfrV</i>	= turbine nozzle optimum lift coefficient fraction
<i>HPTMnexV</i>	= turbine nozzle exit Mach number
<i>HPTNBlds</i>	= number of turbine nozzle blades
<i>HPTNchor</i>	= turbine nozzle chord
<i>HPTNcool</i>	= turbine nozzle cooling air mass flow
<i>HPTNrT</i>	= nondimensional turbine rotational speed
<i>HPTradm</i>	= turbine mean radius
<i>HPTRBlds</i>	= number of turbine rotor blades
<i>HPTRchor</i>	= turbine rotor chord

<i>HPTRcool</i>	= turbine rotor cooling air mass flow
<i>HPTT0in</i>	= turbine nozzle inlet temperature
<i>HPTUrT</i>	= nondimensional turbine blade speed
<i>HPTVa2Va1</i>	= turbine nozzle axial velocity ratio
<i>HPTVa3Va2</i>	= turbine rotor axial velocity ratio
<i>HPTVhubfr</i>	= turbine nozzle hub velocity ratio
<i>HPTwt</i>	= turbine weight
<i>HPT<math>\eta</math></i>	= turbine adiabatic efficiency
<i>NoBlds</i>	= number of compressor blades (rotor and stator)
<i>Rtot</i>	= compressor pressure ratio
<i>TETmaxR</i>	= maximum nozzle outlet temperature for rotor cooling flow
<i>TETmaxN</i>	= maximum nozzle outlet temperature for nozzle cooling flow
<i>totwt</i>	= spool weight (compressor weight + turbine weight)
$\Omega$	= shaft rotational speed

## I. Introduction

THE design of gas turbine engines is challenging. Decomposing by component and discipline makes this complex and closely coupled multidisciplinary design problem manageable. To cope with the magnitude of the design task and the large number of engineering disciplines, the system is decomposed into modular subsystems which are then tackled by specialist design teams [1–3]. Notwithstanding more than 50 years of success, it is worth considering the implications of decomposition on the achievable design performance.

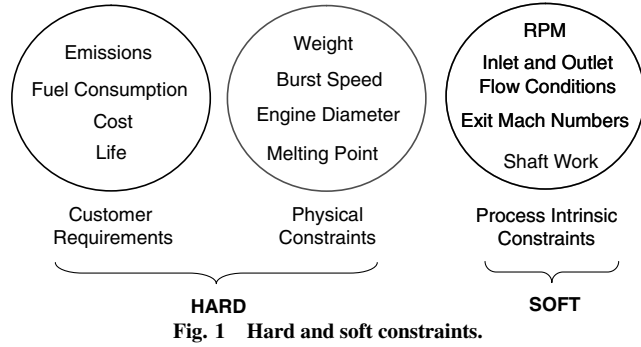
The output of the decomposition is the definition of the input and output requirements for each component or subsystem, which form the core of the preliminary design specification. However, to ensure the compatibility of the components upon assembly, the interfaces between each component and discipline must also be fixed during the preliminary design phase to enable the disciplinary design teams to proceed. This modular approach requires values to be selected for the variables that are shared across the interfaces, including inlet and outlet flow conditions, shaft speeds, component power requirements and bleed air cooling flows. Thus the design specification includes not only performance requirements and physical constraints, but also

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*process-intrinsic* (PI) constraints introduced by the need to specify intercomponent and interdisciplinary interfaces (Fig. 1). The constraints at module interfaces are not “real” constraints in the sense that a material melting point or stress limit is real, but instead are only present because of the decomposition of the design process and are therefore “process intrinsic.” The performance requirements and physical constraints are part of the core design specification and thus are arguably “hard” because they cannot be violated. However, the process-intrinsic constraints that are a direct result of the definition of interdisciplinary and intercomponent interfaces during the decomposition process are not part of the design requirements and as such are arguably “soft”. If the design process could be modified such that the interfaces required less specification, some process-intrinsic constraints could be removed or softened.

Many of the early design decisions about constraints are based on past designs and aerothermal cycle analyses [4–7]. Because decomposition happens early in the design process where decisions will, to a significant degree [8], determine the overall performance, cost, and risk of an engine design, the definition of the preliminary design specification, including the process-intrinsic constraints, is the most critical phase of the gas turbine engine development process [4,5].

The design process has historically been segmented and sequential due to the organizational and computational difficulties of a holistic approach. A fragmented approach makes it difficult to explore cross-disciplinary tradeoffs and assess the system-level implications of process-intrinsic constraints because it relies on the specification of the interactions between subsystems [9]. In the past, designers’ experience has helped to bridge the interdisciplinary and intercomponent gaps in the design process [5,7], but as gas turbine technology matures and new environmental and cost objectives emerge, a more systematic and streamlined approach using multidisciplinary optimization is needed [10–12].

To understand and mitigate the effects of process-intrinsic constraints and other early design decisions on the overall system performance, an integrated multidisciplinary approach is required [4]. With a modular and decomposition-based approach, the component interfaces are problem inputs, but with an integral approach the interface values become outputs [13]. Integration is therefore a vehicle to remove process-intrinsic constraints. Integration provides the potential to postpone decisions about critical interdisciplinary parameters until later in the design process and to better exploit interdisciplinary nonlinearity, both with the aim of achieving higher performance designs [14].

## II. Problem Definition

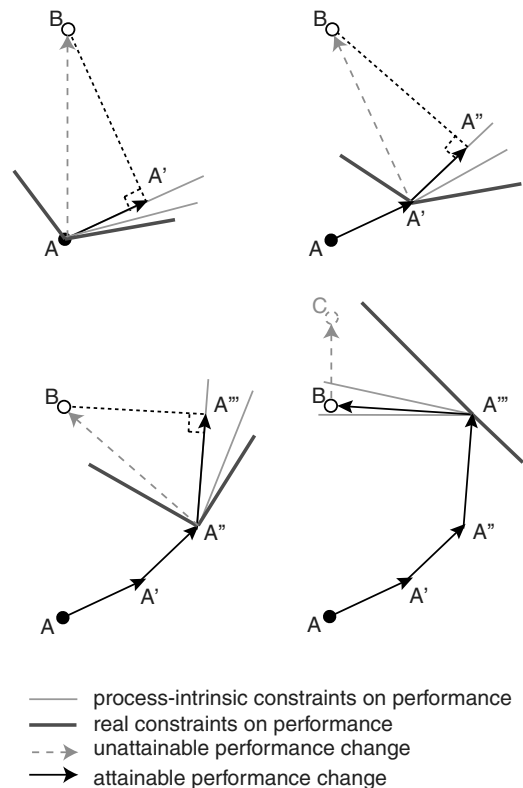
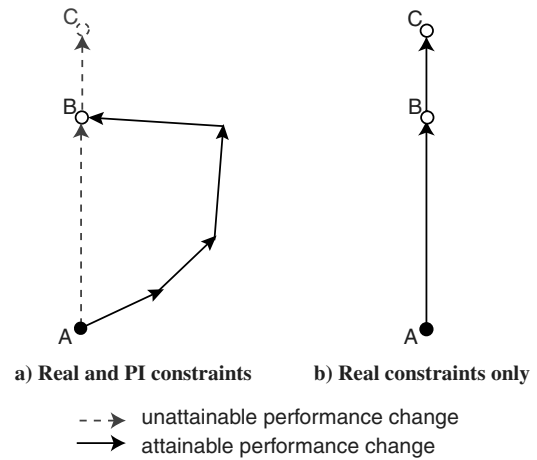
### A. Hypothesis

As a result of decomposition, decisions are made during the preliminary design studies about key design variables that largely determine final system performance. We believe that fixing these variables early in the design process hampers the design process with additional process-intrinsic constraints that are purely a result of the decomposition and are not part of the original design problem. It is our hypothesis that removing process-intrinsic constraints will expand the accessible design space and open up new paths across the

previously constrained design space. This will enable the same performance to be achieved faster and also facilitate higher performance designs that were not attainable with process-intrinsic constraints in place. Softening more constraints should amplify this effect as the accessible design space is further enlarged.

Our rationale for the above hypothesis is demonstrated by the following two-dimensional example. As illustrated in Fig. 2, a performance change from point A to point B with process-intrinsic constraints takes several steps as the direct path is not accessible. By removing, or “softening” PI constraints, the direct route from A to B becomes available and not only is point B attained faster, but the higher performance point C is now reachable.

Figure 3 shows the sequence of design changes required to get from point A to point B with PI constraints. The real constraints, shown with thick gray lines, do not prevent a direct performance improvement from A to B. However, the PI constraints at point A,



**Table 1** Ratio of turbine to compressor efficiency for equivalent influence on SFC

	Walsh and Fletcher	Wisler
+1% HPC $\eta$	-0.78% SFC	-0.66% SFC
+1% HPT $\eta$	-1.20% SFC	-0.82% SFC
HPT $\eta$ /HPC $\eta$	1.54	1.24

shown with thin gray lines, are restrictive and the best possible performance change toward B is achieved by aligning the performance change vector with the PI constraint and proceeding to the point of closest approach, A'. At point A' the real constraints allow a direct approach to B, but the performance is still constrained by the PI constraints with the point A'' being the best possible improvement. Following this process of making the best possible change, point B is reached after four steps.

### B. Performance Metric

The performance metric we will use to measure the effect of removing process-intrinsic constraints is percentage reduction in specific fuel consumption (SFC). By finding the maximum compressor and turbine efficiencies attainable with and without process-intrinsic constraints, we can measure the resulting effect on fuel consumption.

To establish a common frame of reference for comparing the benefits and costs of turbine and compressor efficiency gains, we consider the efficiency gains that will have equal effect on SFC. Although dependent on bypass ratio, flight speed, and altitude, Walsh and Fletcher [15] and Wisler [10] both estimate the percentage reduction in SFC per percentage improvement in high pressure turbine and compressor efficiency. Using their estimates (Table 1), we can calculate their ratios for improvements in turbine and compressor efficiency that will have equal effect on SFC as 1.54 and 1.24, respectively.

### C. Multidisciplinary Coupling

The hypothesis will be tested on a high pressure compressor and turbine spool in a gas turbine engine. A mathematical formulation of the coupled multidisciplinary system is illustrated in Fig. 4. For the problem studied here, there are four disciplinary modules,  $D_1$ ,  $D_2$ ,  $D_3$ , and  $D_4$ , representing the compressor and turbine aerothermal analyses and their respective mechanical analyses. Each discipline  $D_i$  has an associated analysis  $A_i$ , that takes as inputs the system-level design variables  $s$ , discipline-specific design variables  $l_i$ , and any relevant parameters derived from some or all of the outputs from another disciplinary analysis. The compressor mechanical analysis  $A_3$  depends on the blade geometry output from the compressor

aerothermal analysis  $A_1$ . The turbine aerothermal analysis  $A_2$  depends on the power output from  $A_1$  and provides the blade geometry for  $A_4$ . The coupled multidisciplinary analysis (MDA) system is expressed as follows:

$$a_1 = A_1(s, l_1) \quad a_2 = A_2(s, l_2, a_1) \quad a_3 = A_3(s, a_1) \quad a_4 = A_4(s, a_2)$$

The performance parameters  $P_i$  used by the optimizer are subsets of the disciplinary analysis outputs  $a_i$ , as are the coupling vectors  $y_{i,j}$ :

$$P_1 \subset a_1, \quad P_2 \subset a_2, \quad P_3 \subset a_3, \quad P_4 \subset a_4, \quad \text{and} \quad P_5 \subset (P_3 \cup P_4)$$

$$y_{1,2} \subset a_1, \quad y_{1,3} \subset a_1 \quad \text{and} \quad y_{2,4} \subset a_2$$

The fifth performance parameter vector  $P_5$  is present for system-level performance parameters which are dependent on the output from multiple analysis modules and are evaluated by the optimizer and not by a separate analysis module. The target values of the performance parameters for all disciplines are contained in the vector  $P_{\text{target}}$ . The role of the targets is explained in Sec. III.C when the mathematical formulation of the optimization is described.

## III. Methodology

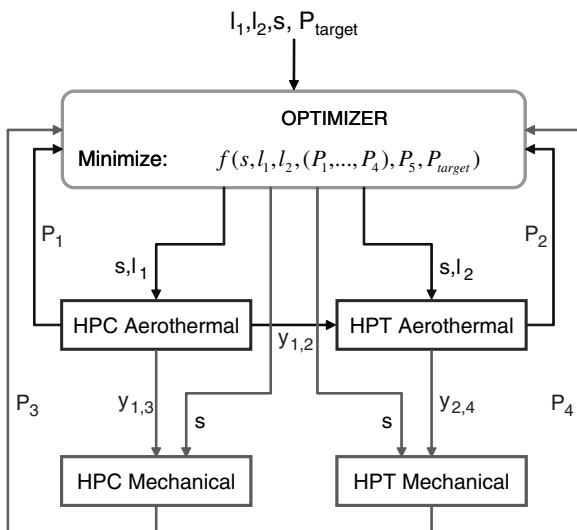
### A. Gas Turbine Design Process

A description of a typical design procedure can be found in many textbooks on gas turbine design [15,16]. Thermodynamic design point studies are used to establish preliminary design specification including estimations of the achievable component efficiencies and weights, which are then passed onto the component specialist design teams. At this point iteration occurs between the disciplinary design teams who each use separate analysis tools to select values for all interacting variables including inlet and outlet flow conditions, shaft speeds, bleed air cooling flows, and power requirements, thus forming process-intrinsic constraints. The design specification is then fixed and passed onto the component detail design teams who are tasked with designing to meet their target performance subject to the constraints set out in the preliminary design. But by the time the detail design phase occurs, it is very expensive to change values fixed in the preliminary design specification because it will require the redesign of many of the components. For instance, if the wrong number of compressor stages or the wrong compressor length has been selected, no amount of optimization at the detail design phase will be able to make up for it [8]. With an integrated approach the design variables in the specification can be allowed to float because their current values can be transmitted to all disciplines at once, maintaining consistency without requiring up-front decisions about process-intrinsic constraints.

### B. Approaches to Design and Optimization of Complex Systems

For complex engineering systems, a more integrated approach has been shown to produce higher performance designs, reduce design time, and require fewer experiments [1,17,18] as well as centralize data storage, improve communication between design teams, and stimulate multidisciplinary trades [19]. The need for an integrated design process has spawned several commercial integrated design environments, including iSIGHT [20,21], FIPER [18], and ModelCenter [22,23], which use parametric models and multidisciplinary optimization. These environments allow designers to link together existing analysis tools and speed up design iterations by automating the manual design process. Undeniably, these tools improve the current fragmented design process through automation, but because the underlying process remains essentially unchanged, they do not necessarily facilitate the removal of constraints [9]. Because many of the challenges for integration are process oriented rather than technology oriented, automation of the analysis codes can only be part of the solution [9].

Integration of the existing design process by automating existing disciplinary analysis tools and placing an optimizer around the automated system should enable higher performance designs. However, we propose that larger gains are possible by using a systems approach where process-intrinsic constraints are no longer

**Fig. 4** Disciplinary analysis coupling.

necessary and interdisciplinary coupling does not need to be specified across physical subsystem interfaces. As integration does require the assembly and automation of disciplinary analysis tools, but in order to remove the need to specify process-intrinsic constraints, an integrated approach that implicitly accounts for intercomponent interactions and nonlinearities is necessary.

Traditionally coupled multidisciplinary systems were solved iteratively to establish equilibrium between the disciplines, or *multidisciplinary feasibility*, which involved repeated solution of the MDA block. The original approach to multidisciplinary optimization involved placing a system optimizer around the MDA and is known as fully integrated optimization (FIO) or multidisciplinary feasible optimization (MDF) [24,25]. However, the computational and organizational difficulties of implementing and executing a fully integrated optimization led researchers to consider decomposition-based approaches that were more aligned with disciplinary boundaries and do not require solution of the MDA [26]. Recognizing the already decomposed nature of the design organization, the use of disciplinary analysis codes and the computational requirements necessary for solving and optimizing the MDA, decomposition schemes and their associated optimization procedures have evolved into a key element of multidisciplinary optimization (MDO) [27].

Decomposition-based approaches to MDO may be broadly divided into two classes: 1) monolithic with a single system-level analysis and optimization or 2) multilevel with analysis and optimization at the discipline and system levels [24,27]. There are many multilevel approaches including global sensitivity equations (GSE) [28,29], collaborative optimization (CO) [30–33], concurrent subspace optimization (CSSO) [34,35], and bilevel integrated system synthesis (BLISS) [36,37]. However, because the disciplinary subproblems are inherently coupled, multilevel and other decomposition-based approaches require the addition of extra constraints and coupling variables to account for multidisciplinary interactions, which for closely coupled systems such as gas turbines can be a significant overhead [24,26,38]. Additionally, although largely successful, there is little analytical proof that multilevel methods are robust or have convergence properties [25,39].

Decomposition by engineering discipline is common and is well suited to company structures with disciplinary design teams. Notwithstanding the above mentioned advantages, design and optimization by decomposition complicates an already difficult problem: decomposition-based approaches reinforce process-intrinsic constraints by maintaining the division between analysis codes and design teams.

Given the highly coupled nature of gas turbine design and our aim to remove the effects of process-intrinsic constraints, another look at the FIO approach is warranted. By integrating all the relevant disciplines and components into a single-level optimization using an FIO approach [25,40], many of the difficulties of decomposition-based approaches can be avoided. No coupling variables or constraints are needed as all multidisciplinary coupling is taken care of implicitly with each discipline having access to all the variables in the system.

### C. Mathematical Formulation

To quantify and explore the effects of removing the process-intrinsic constraints introduced by the decomposition of the design process, we use a monolithic FIO approach that is capable of treating the interdisciplinary coupling variables as either hard or soft depending on whether they are to be present or removed. With a monolithic approach, all the disciplinary design variables remain visible to the optimizer and therefore any interdisciplinary interactions are free to occur, not just those across the specified physical interfaces.

As a result of decomposition, the component design teams are challenged with trying to meet a particular performance specification. Thus, the task facing each team is not the “classical” optimization problem of finding a global optimum, but instead the designers know their desired (i.e., specified) performance target and

they have to find out how to modify the current design such that it will meet the new targets. In recognition and support of this approach, our objective function minimizes a weighted vector norm of the difference between the disciplinary outputs  $P_i$  and their target values  $P_{i,target}$ :

$$f(s, l_1, l_2, (P_1, \dots, P_5), (P_{1,target}, \dots, P_{5,target})) \\ = \|\langle g_i \rangle \cdot (P_i - P_{i,target})_{i=1,5}\|$$

For each discipline  $i$ , the weight vector  $g_i$  is constructed such that each output variable  $j$  contributes to the objective function only if it is outside its allowable range of  $P_{ij,min}$  to  $P_{ij,max}$ . Thus, the weight  $g_{ij}$  is expressed as

$$\langle g_{ij} \rangle = \begin{cases} 1, & \text{if } (P_{ij} > P_{ij,max} \text{ or } P_{ij} < P_{ij,min}) \\ 0, & \text{if } (P_{ij,min} \leq P_{ij} \leq P_{ij,max}) \end{cases}$$

The allowable limits  $P_{ij,min}$  to  $P_{ij,max}$  are expressed as fractions  $c_{ij,min}$  and  $c_{ij,max}$  of the target value  $P_{ij}$  and are a function of whether a given output variable is being considered as a hard or a soft constraint. If  $P_{ij}$  is soft, the allowable ranges are widened by  $2\delta_{ij}$ :

$$P_{ij,min} = c_{ij,min}(1 - \langle \delta_{ij} \rangle)P_{ij,target}$$

$$P_{ij,max} = c_{ij,max}(1 + \langle \delta_{ij} \rangle)P_{ij,target}$$

$$\langle \delta_{ij} \rangle = \begin{cases} \delta_{ij}, & \text{if } (P_{ij} \text{ is soft}) \\ 0, & \text{if } (P_{ij} \text{ is hard}) \end{cases}$$

The purpose of softening constraints is to remove their contribution to the objective function. By setting the value of  $\delta_{ij}$  to be large enough that the variable  $P_{ij}$  always remains with the bounds of  $P_{ij,min}$  to  $P_{ij,max}$ , the weight  $g_{ij}$  is always zero and the constraint no longer affects the objective function. The contributions to the objective function for a hard and a soft constraint are illustrated in Fig. 5. Constraints are not actually removed from the process when they are softened to avoid changing the problem dimensionality. By using the same formulation for both hard and soft constraints, we ensure consistency in the treatment of the optimization problems for each of our constraint softening exercises. This implies that if differences in performance are observed, they are due to differences in behavior and not in modeling.

The design variable vectors  $l_i$  for each component  $i$  can be varied by the optimizer between the limits of  $l_{i,min}$  and  $l_{i,max}$ . If an interdisciplinary design variable  $s_{ij}$  is softened, it is allowed to vary between the limits  $s_{ij,min}$  and  $s_{ij,max}$ . If  $s_{ij}$  is hard, mimicking the situation where it has been determined by an up-front decision, its allowable limits  $s_{ij,min}$  and  $s_{ij,max}$  are set to zero.

### D. Optimization Environment

This approach is embodied by the Visual Integrated Design Space Exploration (VIDSE) environment using the SignPosting methodology [41], which provides a hierarchical database and uses a gradient-based quasi-optimization technique [42] to mimic the requirements-driven design task faced by engine designers. A hierarchical database manages parameter consistency between disciplines and ensures that each disciplinary analysis uses the latest values of disciplinary and interdisciplinary variables.

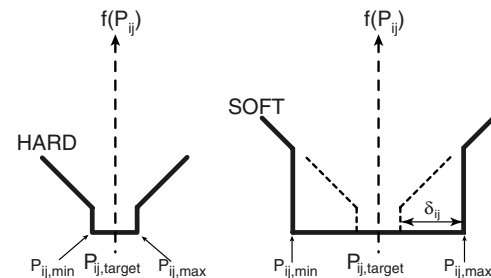


Fig. 5 Objective function for hard and soft constraints.

VIDSE is a modular environment that integrates multifidelity disciplinary analysis codes, proprietary or custom, without modification and interacts with them via text files. SignPosting controls the design variables and constraints, writes the input files, and spawns the analysis modules as necessary. Parameter and task dependencies are used to determine the order of the disciplinary analyses and to capture the systemic effects of design variable changes. Using the concept of smart reanalysis [43], SignPosting establishes which modules need to be recalculated when a given variable is modified and ensures parameter consistency between disciplines [42]. A flowchart of the module execution order for the high pressure spool design task is shown in Fig. 6.

#### E. Constraint Coupling and Sensitivity Calculations

The sensitivities of all performance metrics with respect to each design variable are recalculated at each point in design space by using a simple linearized technique. Each design variable is changed by a small amount and the resulting effect on each performance variable is recorded to approximate the linear sensitivities at each design point. Using linear sensitivities and vector calculations, SignPosting ranks the design variables in terms of their potential to bring the current design closer to the target design specification, that is, reduce the objective function value, and estimates the magnitude of the required change in each design variable. Performance variables which are

within their specified limits are not included in the objective function and therefore do not contribute to the ranking. The ranked design variables are then tested in sequence to see if the suggested change is possible. For instance, the best design variable to change may already be at its upper or lower limit or may be an interdisciplinary design variable currently being treated as a process-intrinsic constraint, in which case the next variable is examined until a change can be achieved. A user-configurable maximum step size parameter is used to limit the allowable change per iteration for the design variables. This has been found to be required for stability due to the nonlinearities of the design space.

Because the sensitivity calculations use all of the disciplinary design and performance variables, the parameter and constraint coupling is implicitly accounted for without the user specifying which variables are likely to affect a given task. For instance, changing a compressor geometry variable can have an effect on turbine aerodynamic performance or geometry as well as affecting the compressor performance and weight. The sensitivities are recalculated at each point in design space such that changes in the active set of constraints are implicitly accounted for. To maintain problem dimensionality, the sensitivity of each design variable with respect to each performance variable is calculated at each iteration, and the weighting factor  $g_{ij}$  ensures that only the active constraints affect the objective function. Thus the method aims to isolate the effect of the constraint softening from the effects of reduced problem complexity.

### IV. Experimental Procedure

#### A. Problem Setup

Using the VIDSE design environment with proprietary compressor and turbine aerodynamic modules and custom weight and combustion modules, we have designed a series of high pressure spools with a range of target compressor and turbine isentropic efficiencies while maintaining the datum overall aerothral performance and spool weight.

The disciplinary and interdisciplinary design variables available to the optimizer are presented in Table 2 and the parameters in the performance specification are detailed in Table 3. Initial values for the design variables, performance targets, and other inputs for the performance specification were selected from an existing high pressure spool design. The discipline-specific performance parameters were maintained within the accuracy of the proprietary software for all cases, but the interdisciplinary parameters were treated differently according to whether they were hard or soft (present or removed) by widening their allowable limits as detailed in Sec. III.E.

#### B. Constraint Softening

The interdisciplinary variables were either treated as hard constraints to emulate their behavior as process-intrinsic constraints or removed in various combinations to investigate their effects on the attainable design specifications and the required design time (number of iterations). For the constraint softening exercise two performance variables and four design variables were softened: component weights ( $W$ ), spool power ( $P$ ), shaft speed ( $\Omega$ ), and turbine inlet temperature and bleed air cooling flows ( $C$ ).

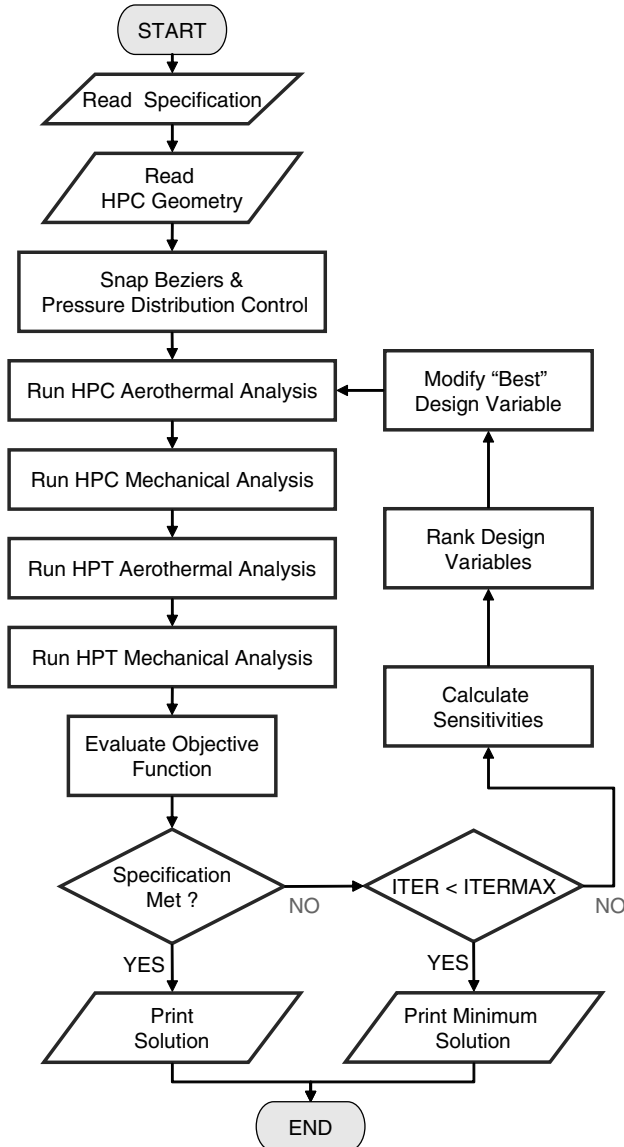


Fig. 6 Optimization flowchart.

Table 2 Disciplinary and interdisciplinary design variables

HPC	Design variables HPT	Interdisciplinary
$l_1 = \begin{bmatrix} bezhub1 \\ bezhub2 \\ bezcas1 \\ bezcas2 \\ dvdp \end{bmatrix}$	$l_2 = \begin{bmatrix} HPTRBlds \\ HPTNBlds \\ HPTradm \\ HPTRchor \\ HPTNchor \\ HPTVa2/Va1 \\ HPTVa3/Va2 \end{bmatrix}$	$s = \begin{bmatrix} \Omega \\ HPTT0in \\ HPTRcool \\ HPTNcool \end{bmatrix}$

**Table 3** Disciplinary and interdisciplinary performance variables

Performance specification		Interdisciplinary
<i>HPC</i>	<i>HPT</i>	
$P_1 = \begin{bmatrix} HPC\eta \\ No\ Blds \\ delr/r \\ Rtot \\ HPCMnPk \end{bmatrix}$	$P_2 = \begin{bmatrix} HPT\eta \\ HPTUrtT \\ HPTNrT \\ HPTANsq \\ HPTMnexV \\ HPTCLfrV \\ HPTCLfrR \\ HPTARV \\ HPTARR \\ HPTalph2 \\ HPTBrel2 \\ HPTVhubfr \\ TETmaxR \\ TETmaxN \end{bmatrix}$	$P_s = P_5 = \begin{bmatrix} totwgt \\ HPCpower \end{bmatrix}$
$P_3 = [HPCwgt]$	$P_4 = [HPTwgt]$	

**Table 4** Hard and soft constraint combinations

	0 Soft		1 Soft			2 Soft					3 Soft				4 Soft	
Case	$S_0$	$S_\Omega$	$S_W$	$S_P$	$S_C$	$S_{\Omega W}$	$S_{\Omega P}$	$S_{WP}$	$S_{WC}$	$S_{PC}$	$S_{\Omega C}$	$S_{\Omega WP}$	$S_{\Omega WC}$	$S_{\Omega PC}$	$S_{WPC}$	$S_{\Omega WPC}$
rpm, $\Omega$		x				x	x				x	x	x	x		x
Weight, $W$			x			x		x	x			x	x		x	x
Power, $P$				x			x	x		x		x		x	x	x
Cooling, $C$					x				x	x	x		x	x	x	x

**Table 5** Interdisciplinary design variable limits (fraction of datum)

Hard constraints				Soft constraints			
$S_{\min}$	$=$	$\Omega$	$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}_{\min}$	$S_{\min}$	$=$	$\Omega$	$\begin{bmatrix} -0.20 \\ -0.10 \\ -1.00 \\ -1.00 \end{bmatrix}_{\min}$
		$HPTT0in$				$HPTT0in$	
		$HPTRcool$				$HPTRcool$	
		$HPTNcool$				$HPTNcool$	
$S_{\max}$	$=$	$\Omega$	$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}_{\max}$	$S_{\max}$	$=$	$\Omega$	$\begin{bmatrix} 0.40 \\ 0.10 \\ 1.00 \\ 1.00 \end{bmatrix}_{\max}$
		$HPTT0in$				$HPTT0in$	
		$HPTRcool$				$HPTRcool$	
		$HPTNcool$				$HPTNcool$	

Sixteen cases were investigated: one with all interdisciplinary variables treated as hard constraints; four with one of  $\Omega$ ,  $W$ ,  $P$ , and  $C$  softened at a time; six with combinations of two of  $\Omega$ ,  $W$ ,  $P$ , and  $C$  softened at a time; three with combinations of three of  $\Omega$ ,  $W$ ,  $P$ , and  $C$  softened at a time; and one with all four of  $\Omega$ ,  $W$ ,  $P$ , and  $C$  softened. Table 4 shows the interdisciplinary variables that were softened for each of the 16 cases. The allowable limits for the interdisciplinary design variables are detailed in Table 5. The performance variables were softened by the amounts  $\delta_{ij}$  in Table 6.

To maintain the problem dimensionality constant for all the cases, all variables were present in all design runs. If shaft speed or cooling was hard for a given case, it was still used for the vector calculations, but was fixed at its datum value with allowable minimum and maximum changes of zero percent. Although these design variables could not actually be modified, they were included in the sensitivity calculations to ensure the dimensionality remained the same between

the test cases. If one of them was selected as the best variable to modify, the next possible choice was used instead. When the cooling flow constraint is softened for the  $C$  cases, the turbine nozzle and rotor cooling airflows ( $HPTRcool$  and  $HPTNcool$ ) as well as the rotor inlet temperature ( $HPTT0in$ ) are allowed to vary. When the component weights ( $HPCwgt$  and  $HPTwgt$ ), are softened for the  $W$  cases the spool weight ( $totwgt$ ) is hardened such that the overall system weight remains the same as the datum weight, but the effect of the constraint is moved farther up the design hierarchy. A softening of the power constraint ( $HPCpower$ ) still means that compressor and turbine power must be equal at all iterations, but that the value can change as the design run progresses.

## V. Results

For each of the 16 cases, over 200 different combinations of compressor and turbine efficiency specifications were considered and more than 4000 spools were designed overall. Each design run was limited to 400 optimization iterations, which took a maximum of 8 min per run on an 1800-MHz desktop PC. Starting with the datum compressor and turbine efficiency, both targets were increased until no more valid designs were being achieved within the iteration limit. Run performance is measured in terms of the minimum objective function value and for successful runs the cost is measured as the number of iterations taken to meet the specification.

**Table 6** Interdisciplinary performance variable constraints (fraction of target)

Constraint softened	Performance variable limits
$W$	$\delta_3 = [HPCwgt] = [0.200]$
	$\delta_4 = [HPTwgt] = [0.200]$
	$\delta_{m,1} = [totwgt] = [0.000]$
$P$	$\delta_{m,2} = [HPCpower] = [0.050]$

### A. Achievable Specifications

The tested specifications are shown in Figs. 7–11 for all the design cases. Each run is represented by a rectangular area with the bottom left-hand corner identifying the target compressor and turbine efficiency and is shaded according to whether or not the specification was met. A shading of black indicates the specification was met with an objective function value of zero, whereas gray shading indicates the specification was not met and the minimum objective function was greater than zero. White indicates specifications that were not tested. For this experiment we were only interested in successful runs that met their efficiency targets, therefore the value of the minimum objective for unsuccessful runs is not considered. Light colored holes in darker regions are caused when the compressor blading correlations fail as a result of changing a design variable too aggressively. Although the system can be configured to recover from correlation failures, we did not use this feature for this study to ensure consistent treatment of all design specifications. Correlation failures typically occur early in the design process when large changes are being made to the design variables and the design is still quite far from its target specification, which results in a more objective function value than for its neighboring runs. Adjusting the maximum step size parameter causes different runs to fail, but the shape of the successful region remains the same.

### B. Benefits of Integration while Maintaining Process-Intrinsic Constraints

Figure 7 shows the benchmark case ( $S_0$ ) with the interdisciplinary process-intrinsic constraints. The datum design is the best specification that could be reached designing the compressor and turbine separately. Integrating the design process while maintaining

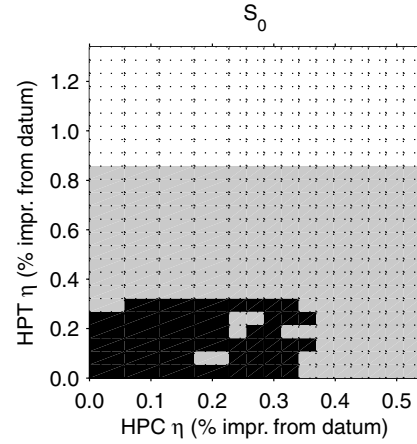


Fig. 7 Specifications met: all four constraints present.

process-intrinsic constraints, improvements in the turbine and compressor efficiencies of 0.27 and 0.34%, respectively, are possible. This illustrates that by integrating analysis modules in an identically constrained design process, such as that available in a commercially available integrated design environment, performance improvements are achievable by exploiting intercomponent nonlinearities.

### C. Effects of Removing Constraints on Achievable Performance

To measure the effects of the process-intrinsic constraints, they were each removed one at a time and then in combination with each

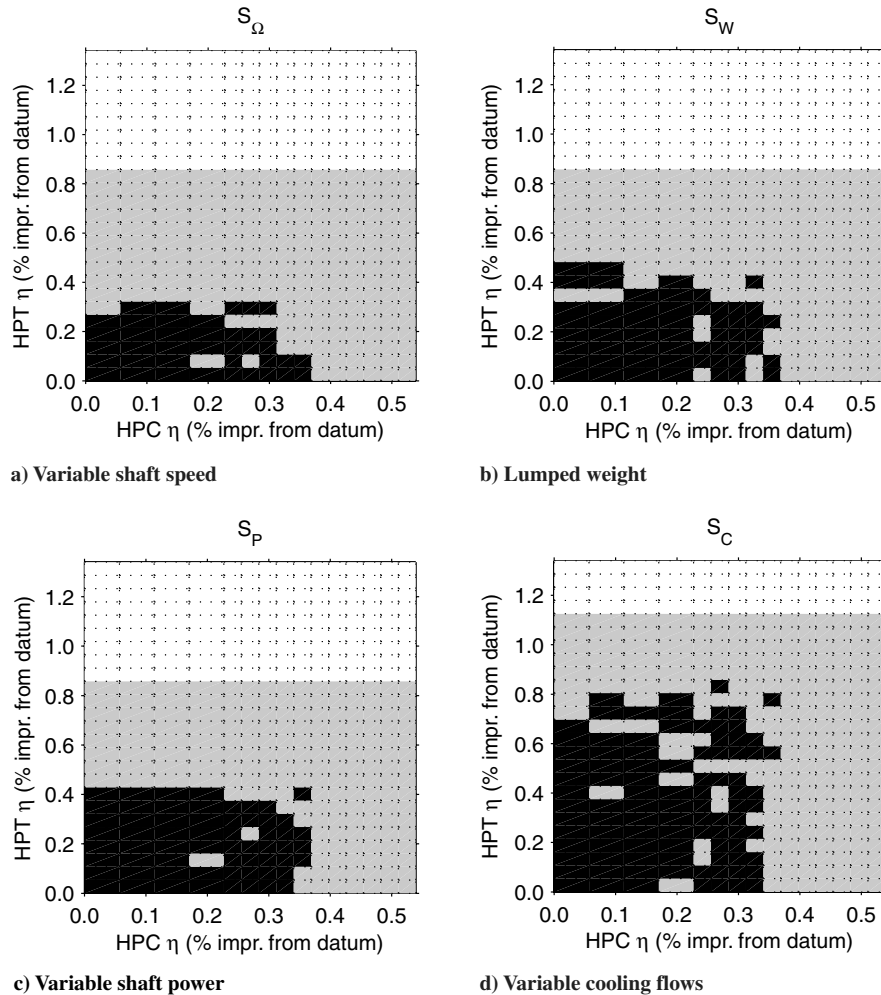


Fig. 8 Specifications met: one constraint removed.

other, by softening their allowable limits, to produce the results in Figs. 8–11. Figure 9 shows the six combinations of removing two of the four interdisciplinary constraints whereas Fig. 10 shows the four combinations of simultaneously removing three constraints. Finally, the results of removing all four of the interdisciplinary constraints are presented in Fig. 11. A comparison of Figs. 7 and 11 illustrates the improvement in achievable efficiencies that result from removing multiple process-intrinsic constraints. Similarly some of the cases with one, two, and three constraints removed also result in

perceptibly better performance than the  $S_0$  case with all four constraints present. Because it is difficult to compare all 16 cases directly, we aggregate the results by the number of soft constraints and can then identify the minimum number of soft constraints to reach a given target. The combined results for each of one, two, and three soft constraints, along with the single zero and four constraint cases, are shown in Fig. 12. These results support the hypothesis that removing process-intrinsic constraints enables higher performance designs to be reached.

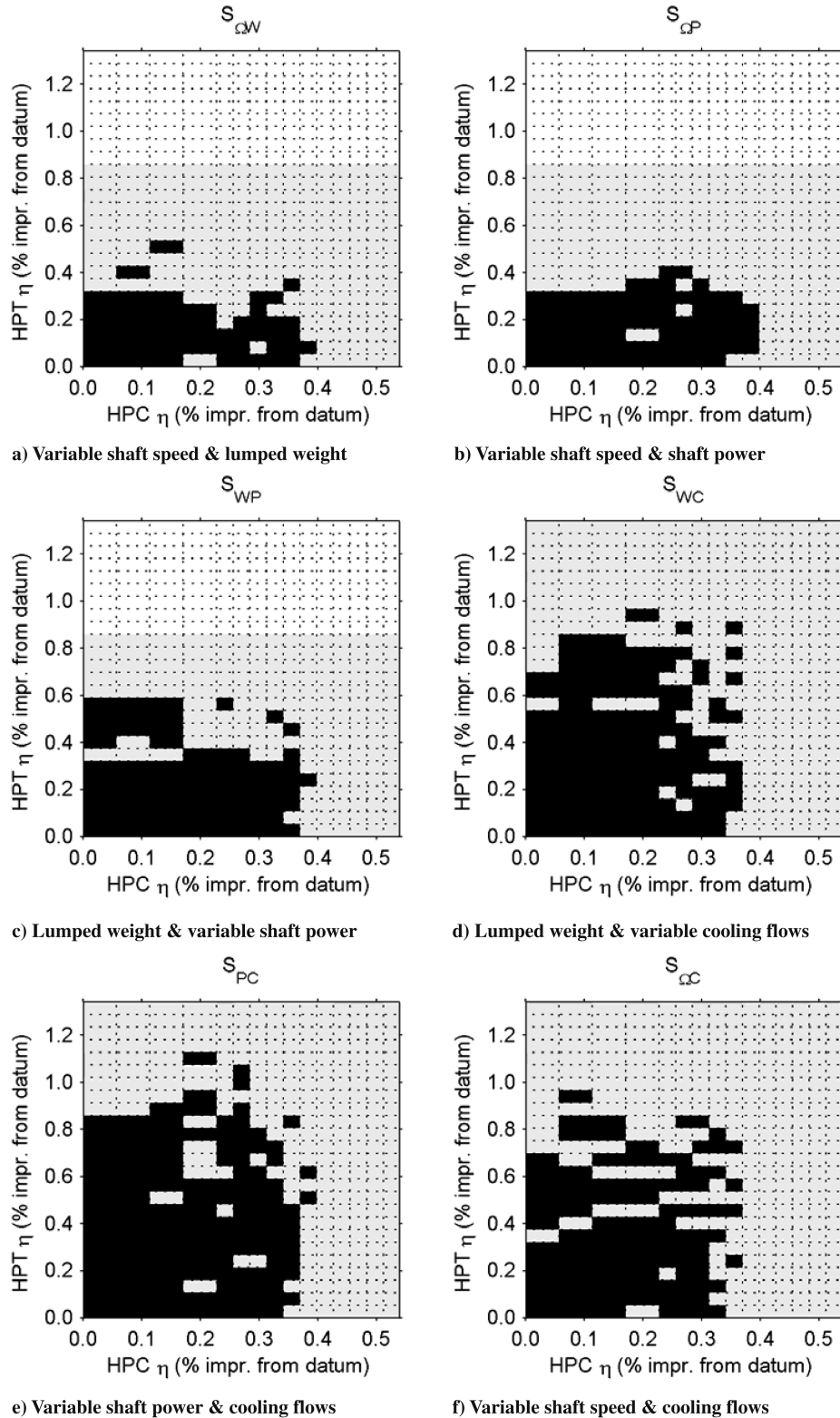


Fig. 9 Specifications met: two constraints removed.



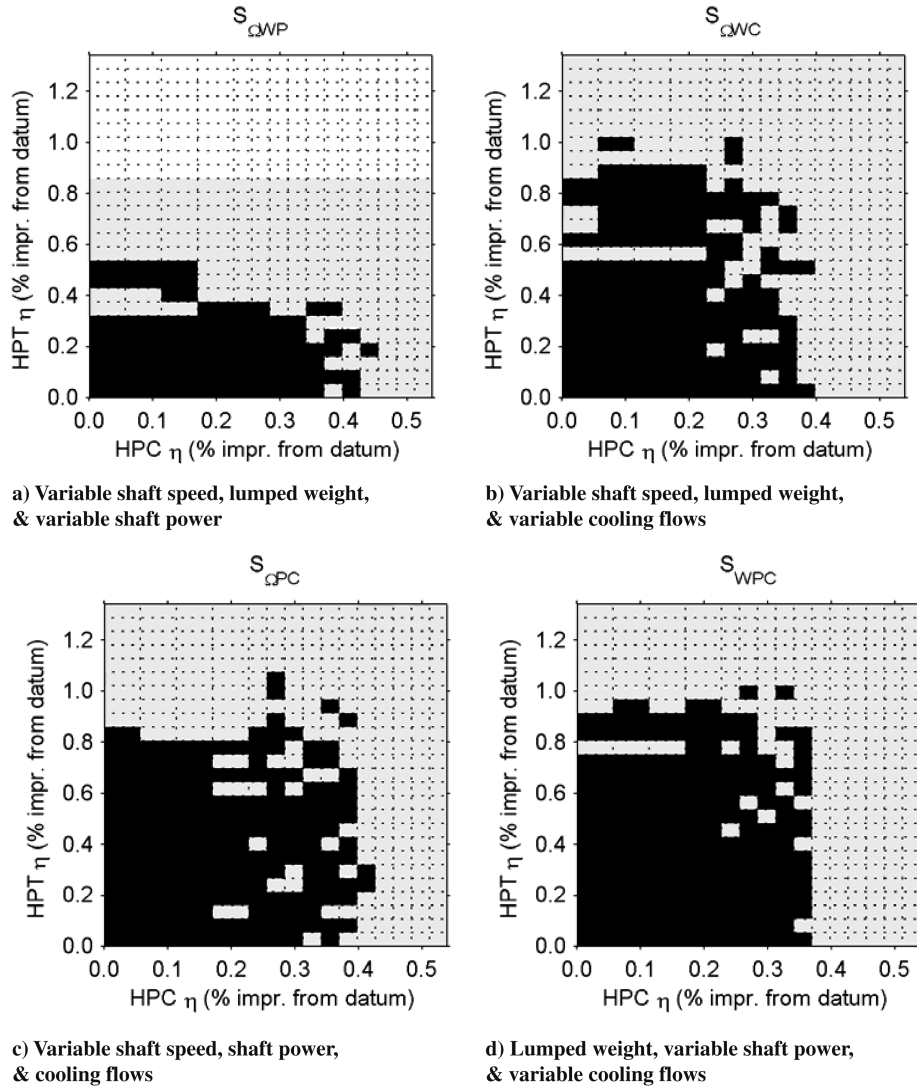


Fig. 10 Specifications met: three constraints removed.

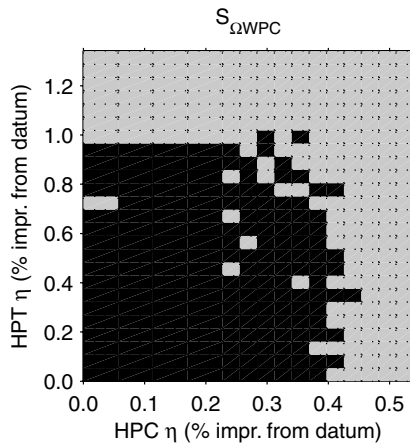


Fig. 11 Specifications met: all four constraints removed.

By assembling the achievable specifications for each number of soft constraints into one plot (Fig. 12e), the benefits of removing multiple constraints are more apparent. When multiple constraints are removed at a time, synergy often occurs with specifications not attainable by removing the same constraints separately becoming accessible. For instance, comparing the specifications that were met with one and two constraints removed reveals that removing a second constraint enables higher efficiency specifications to be reached for

both the compressor and the turbine. These results also support our hypothesis that removing more constraints should result in larger performance improvements.

#### D. Cost Benefit Analysis of Removing Process-Intrinsic Constraints

Although removing more constraints improves the achievable specifications, the results in Fig. 12e indicate that there are diminishing returns for removing more constraints. It therefore makes sense to consider the cost of achieving higher specifications and the relative importance of turbine and compressor efficiency improvements. Temporarily ignoring the desirable ratio of turbine to compressor efficiency improvements, Fig. 13 shows the Pareto front of the best runs and compares the cost benefit analysis of two slices (A–A and B–B) through the design space. Slice A–A is taken through the datum and the design nearest the corner in the Pareto front, while B–B is taken along the line of equal improvement for the compressor and turbine. The cost is measured by the number of iterations taken to meet the specification and the performance improvement is calculated as the square root of the sum of the squared efficiency improvements from the datum design.

The plots exhibit bimodal behavior with one region where modest performance gains can be achieved for a small increase in cost and another region where the cost of further gains increases sharply. Conservative performance improvements of less than 0.3% in Figs. 13b and 13c can be achieved with or without removing constraints for a relatively small change in computational cost. In this

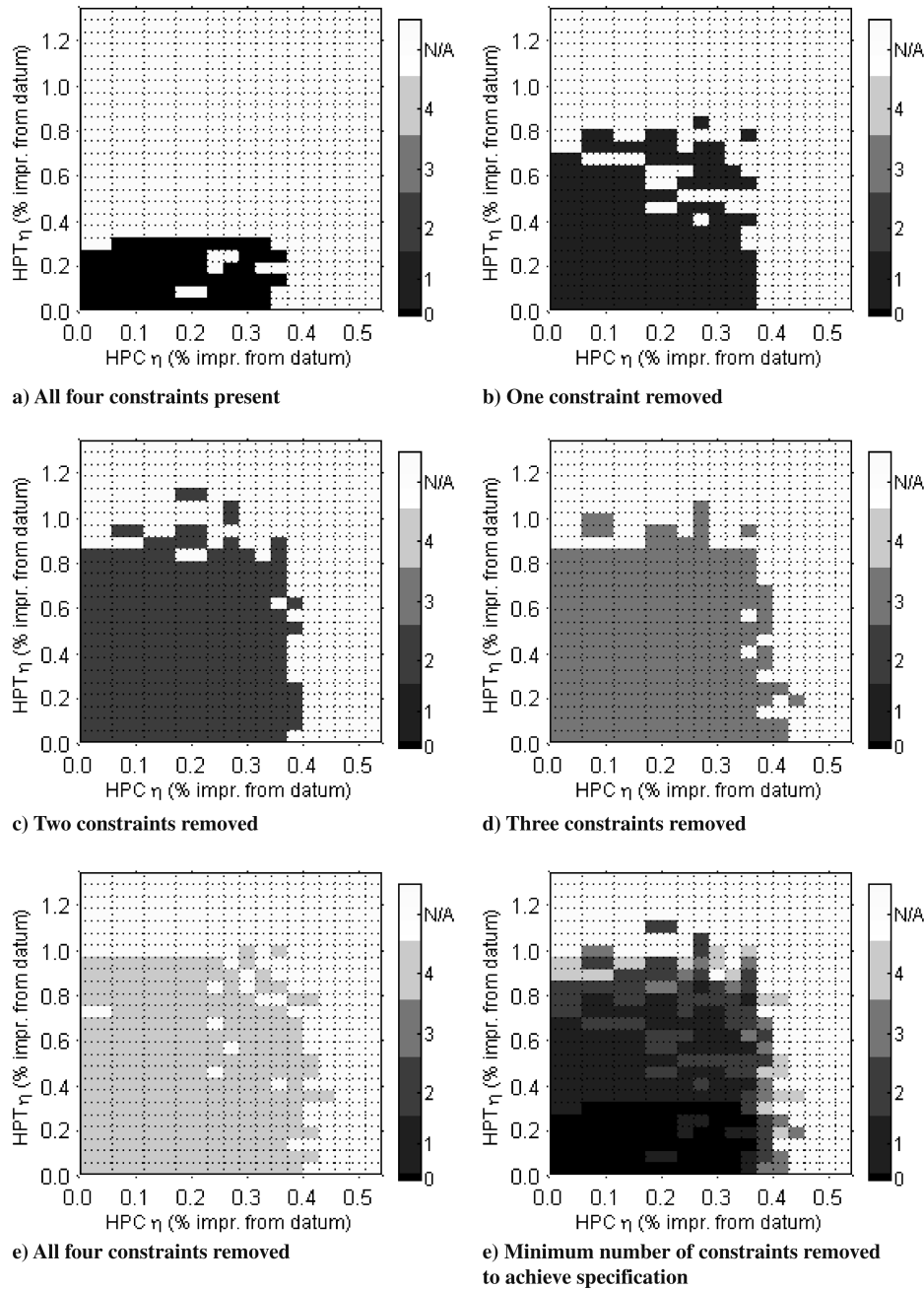


Fig. 12 Specifications met by number of constraints removed.

region, removing more constraints reduces the number of iterations to reach the design specification, but does not change the system behavior. This supports the first part of our hypothesis: removing process-intrinsic constraints enables the same specifications to be reached faster. The second higher performance region is only attainable by removing constraints, but with a significant increase in cost.

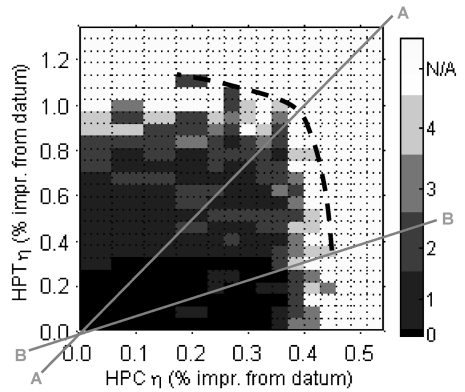
In Fig. 13c, the higher performance designs come with a larger cost penalty than equivalent designs in Fig. 13b, which demonstrates how the desired efficiency improvement ratios can affect the required design time. The advantage of removing more constraints is less apparent for large performance increases with four soft constraints often taking more time than one or two soft constraints.

These trends suggest that constraint softening can be a double-edged sword; it will enable higher performance designs, but with a significant increase in design time. Additionally, the results suggest the optimal number of soft constraints depends not only on the improvement of the demanded design improvements but also on how the required performance changes are distributed between the turbine

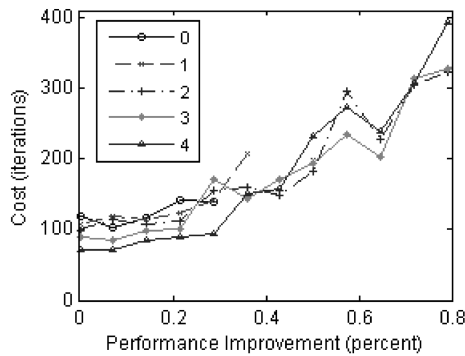
and compressor. This further emphasizes the importance of decisions made early in the design process concerning target component efficiencies and shows that smarter decisions about target specifications may reduce design time.

The slopes of A–A and B–B were chosen relatively arbitrarily ignoring the desirable ratio of turbine to compressor efficiency improvements. If we now use a common frame of reference and consider the relative contribution of component efficiency improvements to SFC reduction, a more appropriate slope can be examined.

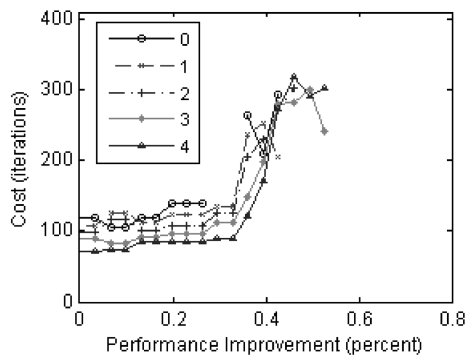
Taking a slice through the datum design using Walsh and Fletcher's slope as established in Sec. II.B, we obtain the results in Fig. 14. Assuming the exchange rate for efficiency improvements and reduction in SFC given by Walsh and Fletcher in Table 1, the corresponding reductions in SFC for removing constraints can be calculated (Fig. 14b). Integrating the compressor and turbine design process, even with the presence of process-intrinsic constraints, can achieve a 0.17% improvement in compressor efficiency and a 0.27% increase in turbine efficiency along a slice through the datum with a slope of 1.54. The SFC exchange rates, however, use percentage



a) Slices through datum and Pareto front (dashed curve)



b) Slice A-A: slope = 3.0



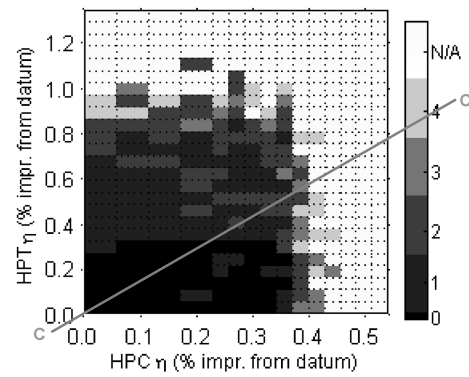
c) Slice B-B: slope = 1.00

**Fig. 13** Cost as a function of performance improvement and number of constraints removed.

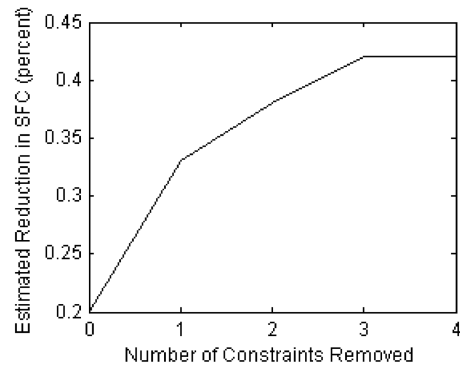
point changes in efficiency; converting the aforementioned percentage improvements into percentage point improvements and using the exchange rates in Table 1, a 0.20% reduction in SFC is achieved by integration alone. However, a much larger benefit of an integrated design process is the ability to remove process-intrinsic constraints, which enables significantly larger efficiency improvements to be achieved. As more constraints are removed, the reduction in SFC increases but with diminishing returns it approaches an asymptote of 0.42% suggesting that there may be an optimal number of constraints to remove. The next phase of our research is to investigate the possibility of determining during the design process which constraints are the most limiting and would provide the most benefit if removed.

## VI. Conclusions

The objective of this work was to use an integrated approach to explore the effects of removing process-intrinsic constraints on achievable design performance. The results support both parts of our hypothesis: removing process-intrinsic constraints reduces design time for already achievable designs and enables higher performance



a) Slice with slope = 1.54



b) SFC reduction along slice

**Fig. 14** SFC reduction as a function of the number of constraints removed.

designs. Integrating without removing process-intrinsic constraints was found to improve performance, but larger gains were obtained when constraints were removed. Having considered 16 combinations with four process-intrinsic constraints, we conclude that removing process-intrinsic constraints produces higher efficiency designs and could therefore reduce specific fuel consumption.

However, the costs associated with these higher performance designs can be significant. A bimodal behavior was observed: for relatively conservative design improvements softening more constraints reduced the cost of achieving the specification though with diminishing returns; larger design improvements were only accessible by constraint softening though at greatly increased computational cost. The relative increase in cost was found to be a function of the desirable ratio of turbine to compressor efficiency improvement. Further study is warranted to determine if the optimum number of soft constraints can be predicted as a function of the desired improvement ratio.

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